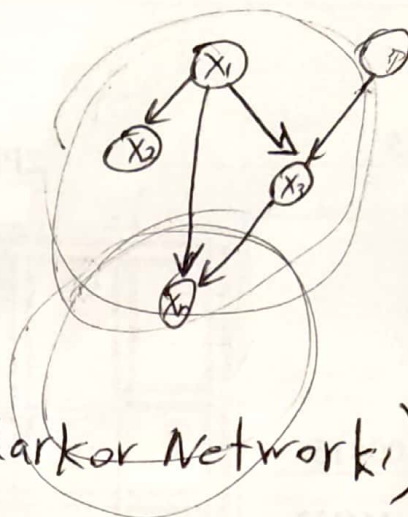


X_1, \dots, X_n

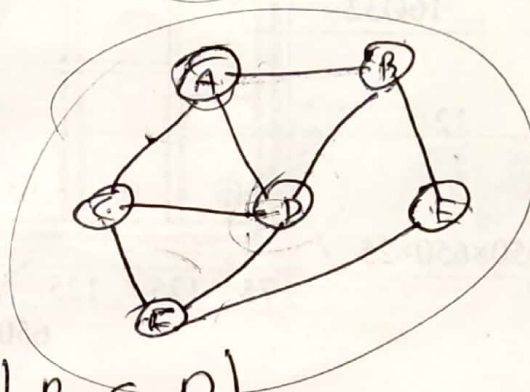
$P(X_1, X_2, \dots, X_n) =$



II

Markov Random Fields (Markov Networks)

$P(X_1, X_2, \dots, X_n)$



$A \perp E \mid B, C, D$

$P(A, B, C, D, E, F)$

$P(A \mid B, C, D, E) = P(A \mid B, C, D)$

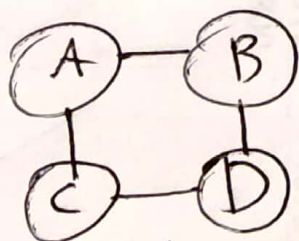
$P(A, E \mid B, C, D) = P(A \mid B, C, D) P(E \mid B, C, D)$

$P(A, E, B, C, D)$

$\sum_F P(A, B, C, D, E, F)$

$\sum_{E, A} P(A, E, B, C, D)$

$\sum_{E, A} \sum_F P(A, B, C, D, E, F)$



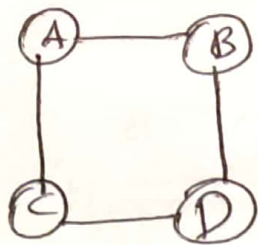
$P(A, B, C, D) = \frac{1}{Z} f_A(A) f_B(B) f_C(C) f_D(D)$

$h_1(A, B) \leftarrow g_1(A, B) g_2(B, D) g_3(C, D) g_4(C, A)$

$\sum_{A, B, C, D} P(A, B, C, D) = 1 \Rightarrow \frac{1}{Z} \sum_{A, B, C, D} f_A(A) f_B(B) f_C(C) g_1(A, B) \dots$

$\Rightarrow Z = \sum_A \sum_B \sum_C \sum_D f_A(A) f_B(B) f_D(D) g_1(A, B) \dots g_4(C, A)$

unnormalized measure $\leftarrow \tilde{P}(A, B, C, D)$



Pg no 6

$$P(A, B, C, D) = \frac{1}{2} h_1(A, B) h_2(B, D) h_3(D, C) h_4(C, A)$$

II

$A \perp D \mid B, C$

$$P(A \mid B, C, D) \stackrel{?}{=} P(A \mid B, C)$$

$$\begin{aligned}
 P(A \mid B, C, D) &= \frac{P(A, B, C, D)}{\sum_{A'} P(A', B, C, D)} = \frac{\frac{1}{2} h_1(A, B) h_2(B, D) h_3(D, C) h_4(C, A)}{\sum_{A'} \frac{1}{2} h_1(A', B) h_2(B, D) h_3(D, C) h_4(C, A)} \\
 &= \frac{h_1(A, B) h_4(C, A)}{\sum_{A'} h_1(A', B) h_4(C, A)}
 \end{aligned}$$

$$\begin{aligned}
 P(A \mid B, C) &= \frac{P(A, B, C)}{\sum_{A'} P(A', B, C)} = \frac{\sum_D P(A, B, C, D)}{\sum_{A'} \sum_D P(A', B, C, D)} \\
 &= \frac{\sum_D h_1(A, B) h_2(B, D) h_3(C, D) h_4(C, A)}{\sum_{A'} \sum_D h_1(A, B) h_2(B, D) h_3(C, D) h_4(C, A)} \\
 &= \frac{h_1(A, B) h_4(C, A) \left[\sum_D h_2(B, D) h_3(C, D) \right]}{\sum_{A'} h_1(A, B) h_4(C, A) \left[\sum_D h_2(B, D) h_3(C, D) \right]}
 \end{aligned}$$